

Ratio solutions for the Gravitation Law.

Recall from math relationships:

$$\text{➤ if } a \propto b \quad \rightarrow \text{ then } \frac{a_2}{a_1} = \frac{b_2}{b_1}$$

$$\text{➤ if } x \propto \frac{1}{y} \quad \rightarrow \text{ then } \frac{x_2}{x_1} = \frac{y_1}{y_2}$$

Therefore, in a two-planet system:

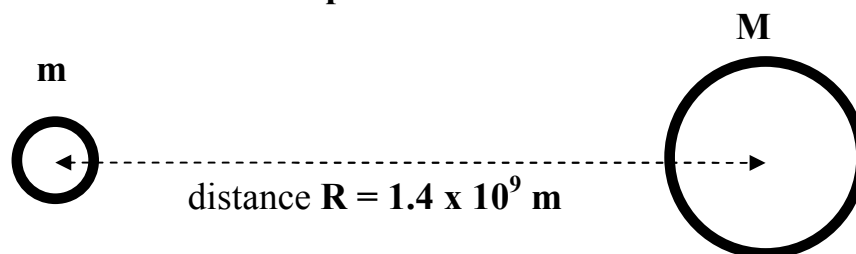
$$F = G \frac{Mm}{R^2} \quad \text{becomes} \quad \frac{F_B}{F_A} = \left(\frac{M_B}{M_A}\right) \left(\frac{m_B}{m_A}\right) \left(\frac{R_A^2}{R_B^2}\right)$$

$$\rightarrow \text{ or, } F_B = F_A \left(\frac{M_B}{M_A}\right) \left(\frac{m_B}{m_A}\right) \left(\frac{R_A^2}{R_B^2}\right)$$

This appears complex, but is in fact based on some very simple math principles:

- if one mass changes, the force changes proportionally; e.g., if the mass of *one* of the two planet doubles, so does the F_g between them.
- if both masses change, the force changes proportionally for each mass; e.g., if the mass of *both* planets double, the F_g between them increases by $2 \times 2 = 4$ times
- if the distance between the two planets changes, the force between them *decreases* by that factor squared; e.g., if the distance between the planets doubles, the F_g between them is decreased to $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Example 5: Examine this two-planet situation:



$$F_g = 4.1 \times 10^{21} \text{ N between the two planets}$$

The above is now changed, as follows; find the new gravitational force in each case:

- (a) m is tripled.
- (b) M is tripled, and m is reduced by half.
- (c) M is one-tenth as large, and R is tripled.
- (d) m is quadrupled, and $R = 9.5 \times 10^8 \text{ m}$.

(see Gravitation Ex 5 for answer)